**Turn in the following problems:**

1. Evaluate your classmate's work computing the following limit. Their work is below:

(i) $\lim\limits_{x\to 2} \dfrac{(x^4-16)^2(1-\cos^2(\frac{x\pi}{4}))}{(\sin(\frac{x\pi}{4}))(x^4-8x^2+16)}$

(ii) $= \lim\limits_{x\to 2} \dfrac{(1-\cos^2(\frac{x\pi}{4}))}{(\sin(\frac{x\pi}{4}))} \cdot \dfrac{(x^4-16)^2}{(x^4-8x^2+16)}$

(iii) $= \lim\limits_{x\to 2} \dfrac{\sin^2(\frac{x\pi}{4})}{(\sin(\frac{x\pi}{4}))} \cdot \dfrac{(x^4-16)^2}{(x^4-8x^2+16)}$

(iv) $= \lim\limits_{x\to 2} \sin(\dfrac{x\pi}{4}) \cdot \dfrac{(x^4-16)^2}{(x^4-8x^2+16)}$

(v) $= \lim\limits_{x\to 2} 1 \cdot \dfrac{(x^4-16)^2}{(x^4-8x^2+16)}$

(vi) $= \lim\limits_{x\to 2} 1 \cdot \dfrac{(x^2-4)^2(x^2+4)^2}{(x^2-4)^2}$

(vii) We have a zero in the denominator since $\lim\limits_{x\to 2} (x^2-4)^2 = 0$. Therefore, the limit does not exist.

If your classmate made any errors in their work, state between which two lines the error(s) occurred (e.g., between line (i) and line (ii)), explain the error(s), and then correctly evaluate

$$\lim_{x\to 2} \frac{(x^4-16)^2(1-\cos^2(\frac{x\pi}{4}))}{(\sin(\frac{x\pi}{4}))(x^4-8x^2+16)}.$$

If your classmate made no errors in their work, then write 'Excellent work, that was a challenging problem!'.

2. Evaluate the limit, if it exists.

$$\lim_{t \to 0} \left( \frac{1}{t} - \frac{1}{t^2 + t} \right)$$

3. Given the function

$$h(x) = \frac{|x - 2|}{x}$$

Determine if $h(x)$ has horizontal asymptotes or vertical asymptotes. Explain your answer using the limit definition of horizontal asymptote and the limit definition of vertical asymptote.

4. A parking lot charges \$4 for the first hour (or part of an hour) and \$2 for each additional hour (or part), up to a daily maximum of \$12.

   (a) Sketch a graph of the cost of parking at this lot as a function of the time parked there.

   (b) Discuss the discontinuities of this function and their significance to someone who parks in the lot.

5. Explain why the function is discontinuous at the given number $a$. Sketch the graph of the function.

$$f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \qquad a = 1$$

6. Show that there exists an $x$ in the interval $(0, 1)$ that satisfies the given equation

$$\sqrt[3]{x} = 1 - x$$

Hint: Create a function $f(x)$ so that you can apply the Intermediate Value Theorem.

**These problems will not be collected, but you might need the solutions during the semester:**

7. Sketch the graph of a function $f$ that is continuous except for the stated discontinuity.

   Neither left nor right continuous at 4, continuous only from the left at -2.

   Explain how your graph satisfies these conditions.

8. Is there a number $a$ such that

$$\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

   exists? If so, find the value of $a$ and the value of the limit.

9. Prove that $\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$.

10. Find the limit.

    (a) $\lim_{x \to \infty} \frac{x + 2}{\sqrt{9x^2 + 1}}$

    (b) $\lim_{x \to \infty} \frac{\sin^2(x)}{x^2}$

11. Find a formula for a function that has vertical asymptotes $x = 1$ and $x = 3$ and horizontal asymptote $y = 1$.

12. (a) A tank contains 5000 L of pure water. Brine that contains 30 g of salt per liter of water is pumped into the tank at a rate of 25 L/min. Show that the concentration of salt after $t$ minutes (in grams per liter) is

$$C(t) = \frac{30t}{200 + t}$$

(b) What happens to the concentration as $t \to \infty$?

***Optional Challenge Problems***

1. Find values of $a$, $b$, and $c$ so that the following function is continuous.

$$f(x) = \begin{cases} 6 - 3bx & \text{if } x \le -2 \\ cx^2 - ax + 4 & \text{if } -2 < x \le -1 \\ 6 - bx & \text{if } -1 < x \le 1 \\ ax^2 + c & \text{if } x > 1 \end{cases}$$

2. Evaluate the following limit. (For those that have had calculus before, feel free to confirm your answer using L'Hôpital's Rule, but solve it another way. This is an algebra challenge!)

$$\lim_{x \to 64} \frac{\sqrt[3]{x} - 4}{\sqrt{x} - 8}$$

3. Below are challenge problems on limits involving infinity. Show and clearly explain all work. Confirm each result you get graphically and/or numerically.

   (a) Find $\displaystyle\lim_{x\to\infty} \frac{\sqrt{4x^2+7x}}{x}$ and $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{4x^2+7x}}{x}$

   (b) Find $\displaystyle\lim_{x\to\infty} (\sqrt{x^2+10x} - x)$ and $\displaystyle\lim_{x\to-\infty} (\sqrt{x^2+10x} - x)$

   (c) Find $\displaystyle\lim_{x\to\infty} (\sqrt{x^2+11x} - \sqrt{x^2+5x})$ and $\displaystyle\lim_{x\to-\infty} (\sqrt{x^2+11x} - \sqrt{x^2+5x})$

   (d) Modify the constants in the function of the last problem so that the horizontal asymptotes lie at $y = \pm 15$.